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Articoli e saggi

The termination for the user's breach in the leaseback: an example of mathematical analysis of the law

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1. *Introduction*

The paper contains a dialogue between two disciplines: law and mathematics.

Legal certainty and mathematical certainty have always had a common denominator: «the reports enucleated sensitive intuition, even if the fundamental institutions of the postulates of the two sciences have been distilled through abstractions, often very high»[1].

Surely, law began as a descent from cosmic harmonies in the concreteness of *civitas* and their conflicts through an interpretation that is based on the proportion, in its long history of secularization. Law retains its reference to the balance of interests or protected goods, the resulting measure of sacrifice and powers in the construction of relationships. Even in the formalism, this idea does not lose because, just to form, it is attributed the role of creating measurement and bulwark in social relationship. It is from very ancient roots that among the general principles of law, unwritten, they include reasonableness and proportionality.

This development could become useful tool in the relationship between mathematics and law: the logos of proportion, looked through the mathematical tools.

[1] So, A. Flores, *Proiezioni del diritto nella logica matematica. Il provvedimento d'urgenza, in informatica e diritto,* Firenze, 1982, 27.

Lengthily, reinterpreting legal rules through the support of the mathematical logic has been taken under advisement as a crucial and necessary point. That requirement was met in 1981 in the international conference on «Logic, computer science, law» held in Florence. The belief that emerged in perfoming the work was that every juridical problem must canonically have a narrow juridical solution in the vice of logic: only the logically and formally juridical solutions can form a case law coming from the certainty of the law, as such elevating to the degree of the scientific search.

In spite of the scarce application in the coming years of reported beliefs, there is no documentary evidence able to point one grown sensibility and awareness up in order to the combination law-math, which could bring to a fertile evolution of law, especially in the economic aspects of the juridical institutes.

Without presumption of completeness, it should be taken into consideration the utility and the role played by mathematics for the resolution of legal issues, particularly considering a statistics-probabilistic approach.

For example, the historical problem of Cardinal Newman, regarding the weight of the circumstantial evidence in the decision of a judge, to define a threshold that, once exceeded, guilt is certain. And still, the quantitative methods and the financial mathematics develop an important function turning out to be useful in case of a stipulated contract.

The present paper considers the hypothesis of the rescission of contract for user's breach in the leaseback, with particular reference to the contractual clause which includes the determination of the credit of the leasing company in reason for the sum of the general actual value of the due payments and of the ransom price, besides the restitution of the good.

The leaseback concerns a good intended to retain significant value at the end of the contract, so that the user's interest is presumably to buy ownership of the good, by exercising the purchase option after paying all fees agreed[2].

The importance of that distinction is that, according to a case law interpretation, the article 1526 c.c.- whereby, in the event of rescission of the contract is done for buyer's breach, the seller is obliged to return the installments collected, but is entitled to be given a fair compensation for the use of the thing, in addition to damages – applies only to leaseback, and not to leasing.

In the leaseback only the user's interest is to become owner of the asset at the expiry of the contract, and the canons have the function of anticipation instalment of purchase price[3].

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[2] That distinction has often been reaffirmed by the judge of legitimacy. So, Cass. Civ., 28 August 2007, n. 18195.

[3] Cass. Civ., 10 September 2010, n. 19287; 8 February 2010, n. 73; 9 June 2008, n. 1969.

Almost all leasing companies in Italy adopted the termination clause «expired + expire + good»: in hypothesis of rescission of breach contract, the user must pay all amounts owing for overdue fees and not met, as well as pay the fees not expired yet and the redemption price, in addition to the return of the good, to a penalty.

This clause leads to an unjustified enrichment of lessor, which follows more than they would have been entitled to obtain in the case of regular fulfillment of the user. So, also, in marked contrast to the prediction of art. 1383 c.c.

The case law has gone up to the ruling of illegitimacy of the expressed resolutive clauses of lease agreements, not in compliance with the dictates of art. 1526 c.c., which was proclaimed the mandatory nature[4]. The convinctions led the lessor to the adoption of expressed termination more balanced, such as one called «expired + expire- good»: against the rescission of contract for default, due to the payment of all rental accrued and accruing until end of the leasing contract, redemption price included, but also the right of the user in default to get the financial consideration for the sale or re-lease of the good.

In this case, the lessor would not have more than they would have been entitled to obtain, in case of regular fulfilment of the user. That, in accordance with the dictates of art. 13 of the Unidroit Convention of 26.5.1998 in Ottawa, on international financial leasing, according to which the penalty provided for user's default must be sufficient «to put the lessor in the same situation in which he would have found if the user had exactly fulfilled to the leasing contracts».

More recently, the Supreme Court Justice judgement n.888, 17 of January 2014 has brought back the attention of many questions regarding the termination clauses contained in leasing contract, and in particular a probable conflict with article 1526 c.c.

Once qualified the clause as a penalty and considered the same as manifestly excessive, the Suprem Court has ruled that, in order to avoid the excessive nature of the benefit to the lessor, it is a prerequisite being provided by a contract or the user's attribution to the right to recover in precise timing the ownership of the good – when returned the entire amount of funding -, or the right to impute the value of property to the sum due in repayment of expire rates.

The legitimacy of the clause «expired + expire – well» excluded the excessiveness of the penalty in the event that the lessor had limited its question to the difference between the credit and the presumed realizable value of the good already leased [5].

[4] For example, Cass. Civ., 27 September 2011, n. 19732.

[5] Trib. Treviso, 19 May 2014.

It was so well established the legitimacy of the termination clause «expired + expire - well», on condition that it does not provide wide discretion and only generic obligations for the lessor.

The recent case law seems to have been expressed without taking into consideration the wear discipline, or rather, without studying carefully the impact that this new clause could have had in defining the contractual interests, as we will show later in this paper.

This paper addresses a hermeneutical and mathematician path on the application in practice of legislation against wear.

This combined work will attempt to prove the use of the disciplines of mathematics and law. The contractual interest rate, in the case of the activation of the clause, is different from the interest rate detectable once subtracted the value of the sale of the good by the due credit.

The proof of that assertion leads to the demonstration of how the subtraction from the credit from value of good's sales is actually an indirect circumvention of the wear's rules of the interest rates.

For the purposes of this demonstration, the mathematical finance will assist at the legal speculation, through an analytical study of the case.

The problem reported, supported by analytical mathematical demonstrations, requires legislative intervention to restrict the contractual freedom of leasing companies, in the forms.

So, in the firm conviction of the indispensable knowledge of rational tools, such as statistics, probability and financial mathematics to the purpose of an effective application of law.

2. *The meaning of the term lease from its origins to the European experience*

The evolution of the model contract – in the absence of a legal discipline – led to a sufficient degree of typing of contracts prepared by leasing companies[6] .

The Supreme Court has considered the leasing «negotiating practice by which a leasing company allows at an economic operator the enjoyment of a good, behind the consideration of a fee for a certain period of time»[7].

In light of this, the discipline of leasing is the result of the processing of the interpreters; very often, the evolution of contractual practice was in breach of obligatory rules, as we will try to prove in the course of treatment.

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[6] G.F. Campobasso, *Diritto Commerciale. Contratti, titoli di credito, procedure concorsuali*, Milano, 2012, 147.

[7] Cass. Civ., 30 June 1998, n. 6412, in *Danno e responsabilità*, 1998, XI, 1044.

The term leasing appears the most appropriate to the identification of the contract, especially with a view to overcoming the narrow national borders.

In this regard, it should be noticed the formation of a process of denationalization of the contract, determined, either important regulatory competition between different legal systems, or from instances of unification concerning especially the contractual private law. It is essential the comparison with other languages[8], expression of such models.

The choice to use the English language for the present work responds to these principles, as well as on the transnational significance of the discussion topic. The lease is now an instrument of action of economic agents in the context of the markets.

It is appropriate to observe the institute also in a supranational perspective[9] , for the multiplicity of sources that help regulate at national and international level[10]. So, in order to contribute to finding a uniform law, valid at the international level; as well as the creation of conventional rules that are perceived as binding by operators far beyond national borders.

In support of this, refers to a new *lex mercatoria*[11] sets from the practice of farmers that using uniform rules in the global market; as well as the establishment of a global code of international trade[12].

Despite the unrealized idea of a European private law – particularly with regard to company contracts – registering a process of harmonization of law by the community legislation, which aspires to become true unification of law by a european civil code.

It is however important to note that a totally uniform contract law does not exist. In addition, the harmonization concerns the contracts with consumers, so that only a small portion of leasing contracts apparently hit by such movement of harmonization. Otherwise, business contracts usually evade any ambition of harmonization and unification.

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[8] In support of this, A. Ortolani, *Le lingue del diritto. Nuove prospettive in tema di traduzione e interpretazione del diritto plurilingue*, in *Riv. Crit. Dir. Priv.*, IV, 2003, 203.

[9] In argument, A. Frignani, *Il Leasing negli ordinamenti di civil e di common law*, in *Riv. it. leasing*, 1998

[10] See L.J. Costantinesco, *Il metodo comparativo*, Italiana ed. by A. Procida Mirabelli di Lauro, 299 ff. For an analysis of the relationship between comparative law and international law, community law and uniform, R. Sacco, *Introduzione al diritto comparato*, Torino, 1992, 154 ff.

[11] V. Buonocore, *Contrattazione d'impresa e nuove categorie contrattuali*, Milano, 1997, 192 e 205 ss.; see also F. Galgano, *La globalizzazione nello specchio del diritto*, Bologna, 2005.

[12] M.J. Bonell, *Comparazione giuridica e unificazione del diritto*, in Aa.Vv., *Diritto privato comparato*, Roma-Bari, 1999, 3 ff.

3. *The rescission of contract for the user's breach in the leaseback and wear rules*

The rescission for breach needs a distinction: the breach of the lessor must be treated separately from the user's breach. In almost all leasing contracts, there is the presence of clauses which transfer all risks of the contract by the lessor to the lessee: falls on the user – rather than the lessor – the liability for failure to deliver, for destruction, for defects, lack of quality requested for the eviction.

The most recent case law has shown that the transfer of risk from the lessor to the lessee, by exception, became rule: in some judgments, the inversion of responsibility is considered natural[13] and structural element of finance leases[14] .

It refers, in general, to the contractual clauses providing that the leasing company is entitled to fees paid[15] together with default interest, as well as to ask the complete restitution of fees with two variations: pay only a percentage of the fees not expired yet and the deduction of the price of the good. It is useful to specify how the leasing company can freely define the price considered cheaper and the user has no right to intervene.

This discussion analyzes, in particular, the possibility of termination for breach in leaseback, whose compensation may be previously established under art. 1382 c.c.[16], explaining the lawfulness of that agreement, as an expression of private autonomy. Therefore legitimate, but not necessarily fair just for the fact of being sued, because the art. 1384 c.c. allows the judge to reduce equally the provision[17].

This work, through the combined use of the disciplines of mathematics and law, tends to show, that the contractual interest rate, at the activation of the clause, is different from the interest rate detectable once subtracted the value of the sale of the good by the due credit.

Said hermeneutical and mathematician path tends to protect from wear. The law 108 of 1996[18] has deeply revisited the crime of usury governed by the previous drafting of article 644 c.p. and exacerbated civil consequences stated in

[13] For example: Trib. Milano, 27 June 1985; Trib. Milano, 19. September 1985; Trib. Torino, 5 September 1985.

[14] So, V. Buonocore, *La locazione finanziaria*, Milano, 2008, 184.

[15] In favour of the application of art. 1458 c.c., resulting in retention of rate precette, are, for example: Cass. Civ., 6. May 1986, n. 3023 in *Foro it.*, 1986, and *Riv. it. leasing*, 1986; Cass. Civ., 26 November 1987, n. 8766, in *Giur. it.*, 1988, and *Foro it.*, 1988; Trib. Milano, 6. December 1990, Giur. Comm., 1991, II. On the contrary, they are for the full implementation of art. 1526 c.c. with a requirement to return the collected rate, Corte App. Milano, 14 April 1987, in *Riv. it. leasing*, 1987; Trib. Milano, 16 May 1988, in *Riv. it. leasing*, 1988, Trib. Vicenza, 1 July 1988, in *Riv. it. leasing*, 1989; Trib. Milano, 26 September 1988, in *Giur. Comm.*, 1989.

[16] In favour, Cass. Civ., 24 June 2002, n. 9161 in *Arch. Civ.*, 2003.

[17] So, V. Buonocore, *La locazione finanziaria*, cit., 184.

[18] For an analysis of the elements of novelty in preventive key of law n. 108 of 1996, see, among others, P. De Angelis, *Usura,* in *Enc. Dir. Treccani*, XXXII, Roma, 1997.

art. 1815, paragraph 2, c.c. by replacing the original mechanism of usurious rate reduction to legal measures.

In the rule of law, the transition from a contested prevision that did not define the usurious rate, to that defined – already in general and abstract – changes the characteristics of the crime of usury and the structure itself, going from damage offense or better real danger offense[19], to crime of abstract danger or alleged.

It was in fact introduced the so-called objective wear that is realized when it was agreed an overflowing rate-threshold in force at the time of completion of negotiations, according to the procedures of articles 2 and 3 of Law 108/96. It was then set at numerical measure and objective which, for each quarter, the upper limit not to be exceeded because the contract does not arise from the origin usuriousis.

The second part of paragraph 3 of art. 644 c.p. however also has a subjective parameter that qualifies as usurious «those interests which, although lower than the threshold rate, are still disproportionate to the consideration of money or other benefits, in the event of economic or financial difficulties»[20].

These residual cases, are aimed to fill possible gaps in protection, implemented through performance at an agreed interest, below the legal limit set to evade the precept[21] .

The judge enjoys wide discretion in declaring usurious interest below the threshold rate. This, quenching the automaticity of the objective principle of the legal limit referred in the first paragraph and repeats the subjective principle of the economic conditions or financial difficulties, already present in the previous version of article 644 bis c.p.

The new law of '96 has introduced a different size to compare with the rate-threshold to assess the lawfulness of the financing. In fact, if the first paragraph of art. 644 c.p. does a generic reference to the term «interests», in the second subparagraph, introduced precisely by law 108/96, is defined as the term should be understood for the purposes of wear the phrase «interest»: «For the determination of usurious interest rate, account shell be taken of committes, remuneration in any capacity and expenditure, except for fees and taxes, related to the provision of credit». If, on the surface, the term «interests» seems to be the same used in the previous text of article 644 c.p., in substance, has been made an epochal revolution.

The new interest rate described by the legislator, to confront with rate-threshold is the APR, the total cost of the credit, net of fees and tax and taxes, in which

[19] The Supreme Court ruled to that effect in its judgment n. 11837, of December 10, 2003.

[20] A. Turco, *Il tasso soglia usurario e il contratto di mutuo*, in *Riv. Notariato*, 2, 2005, 265.

[21] See, S. Prosdocimi, *La nuova disciplina del fenomeno usurario* in *Studium iuris*, 1996, 598.

the interests not detect in their nominal size, but only by virtue of generating cash flow together with all other items of cost of financing.

4. *The threshold rate and the APR*

The crime of usury is implemented in the presence of objective factors of reference, if a consideration contract, the consideration exceeds a certain threshold[22], set by the legislator, beyond which interest must always be considered as usurious[23].

The notion of interest objective, adopted by law no c.d. 108 of 1996, is characterized for greater extension, in order to prevent the easy evasion of the new legislation, which could be implemented by money allocation for miscellaneous expenses, rather than capital[24].

This definition opened a broad debate[25] and sparked several jurisprudential contrasts, «repeatedly acknowledged by legislative measures[26] of doubtful effectiveness and opportunities»[27].

The definition of APRC also includes the utilities, connected in various ways to the expenditure of credit, legitimizing the distinction between nominal interest rate – equivalent to the simple consideration for the enjoyment of money – and effective interest rate or real – that is the total cost for access to credit.

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[22] In argument, M.G. Siliquini, *Camera dei Deputati (Centro Studi - Documentazione e ricerca)*, *Disposizioni in materia di usura* - Lavori preparatori 7 Marzo 1996 n. 108, XIII Legislature, Roma, 1996.

[23] In argument, I. Caraccioli*, Il reato di usura e le sue possibili connessioni con il credito bancario ed interfinanziario*, in *Il fisco*, 1997.

[24] V. Pandolfini, *Gli interessi usurari*, Milano, 2002, 93.

[25] See, P. Dagna, *Profili Civilistici dell'usura*, Padova, 2008.

[26] With d.lgs. August 4, 1999, n. 342, the Government had intervened in a restrictive manner to the concept of usurious interest in accordance with the l. 108/96 and, in favor of banks, against a case law of the Supreme Court which was forming (See, in particular, Cass. Civ., 16 March 1999, n. 2374 and Cass. Civ., 30 March 1999, n. 3096, in *Corr. Giur.*, 1999, 561 ff., for the exclusion of the use of a legal nature of the quarterly capitalization of interest expense (so-called compound interest), introducing a kind of amnesty for the interests of current and capitalizations already taken place. The Constitutional Court, with ruling of 17 October 2000, n. 425, stated the rule in art. 25, third paragraph D.lgs. n.342/1999, unlawful for excessive delegation. With d.l. 29 December 2000, n. 394, rule of authentic interpretation of law 108/96, the Executive had, therefore, ordered that «the purposes of art. 644 c.p. and art. 1815 seconds paragraph of the Civil Code, are considered usurious interest that exceed the limit established by the law when they are pledged or otherwise agreed, for whatever reason, regardless of the time of payment». It was introduced, thus, an amnesty for the benefit of the banks, for a variety of reports usurious loan, without distinguishing between functional defects of the loan and between time of signing the contract and contents. Thus P. Dagna, *Profili civilistici dell'usura*, cit.

[27] P. Dagna, *Profili civilistici dell'usura*, cit., 42.

«In this regard, the APRC is essentially coincident with the APR introduced by the banking law in the context of the rules on the transparency of banking contracts, in particular for consumer credit»[28].

The third paragraph of art. 2 of the law provides for the obligation to affix in their seats and dependencies open to the public notices containing the classification of financial transactions and the disclosure of rates provided for by ministerial decree for banks and financial intermediaries. The rule, aimed at ensuring a complete and comprehensive information, does not provide sanctions in cases of obligation breach[29].

The formula of financial mathematics to calculate the APR[30] is that of IRR (internal rate of return), expressed in DM 08.07.1992 and transposed in the instructions of the Bank of Italy.

It represents the measurement of financial flows (expressed in absolute terms and determined both temporally and quantitatively) generated both of the costs that of the interests, in percentage terms according to the prediction of art. 644 c.p. and by law 108/96.

The cash flows, translating into a common language the costs (normally expressed in absolute terms) and interests (normally expressed as a percentage), allow measurement mathematics of the cost of a financing, which must be taken into account factors expressed in non-homogeneous form.

The IRR, for every possible hypothesis adjusted contract, allows to calculate the Effective Rate Global through the reshaping of the original amortization schedule agreed, on the basis of evolutionary scenarios in terms of loan flows repayment.

It is possible to develop a specific amortization schedule relates to any situation envisaged and regulated by contract and measure, for each of them, the APR of credit granted and disbursed. This rate represents the parameter which the law 108/96 recognizes a synthetic and comprehensive value to verify the wear.

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[28] V. Pandolfini, *Gli interessi usurari*, cit., 94.

[29] See., E. Gianfelici, F. Gianfelici, *Le misure contro l'usura-banche e tassi usurari*, Milano, 2004, who they believe that, returning the threshold-rate in the concept of interest in art. 116, first paragraph, D.lgs. n. 385/1993 (TUB), would still apply, even in the absence of express reference, the administrative fines provided for in the field of advertising required to be given to interest rates, prices, the communication costs to customers and all other economic condition concerning the operations and services offered, including default interest and currencies applied for the charging of interest.

[30] Must be computed in the APR only the cost elements emerging in the stipulations contractually. Once learned from the contract cost elements, it is possible, through the IRR, exactly quantify from the time of the negotiation processing, which will be the APR, or the overall cost of the financing, in all possible scenarios that could potentially develop on the basis of contractual conditions at a given moment in history future. In the opposite direction, authoritative doctrine calls for the affirmation of the principle of inclusiveness in the interest rate for wear, in deference to the correct methodology introduced by the reform of '96. In argument, R. Marcelli, *Oneri eventuali, interessi di mora e penale estinzione: la verifica dell'usura dettata dall'art. 644 C.P. ha un solo criterio di calcolo: il rendimento effettivo*, in *www.assoctu.it,* June, 2015.

The law sets an absolute and binding limit on the costs and interest envisaged in the contract, regardless of the corresponding nature, compensatory, moratorium or criminal, and this limit must be subjected contractual terms and conditions.

With the replacement of the APR (total cost) to nominal rates which size to compare with the rate-threshold, the only way in which the default interest can have an impact on the sieve usurious is the calculation of the effects of financial flows on the APR, together with the other cost items; not however as «dry» comparison between the rate at which are expressed and the rate-threshold or, worse, creating a false as arbitrary specific wear threshold.

Therefore, even an the implementation of an interest rate initially higher than the rate-threshold is not sufficient to the affirmation of the wear rate, if not have measured the effects on the APR along with all the other costs and interests.

It can be concluded, therefore, that the discriminating between a usurious contract and legal is represented by the comparison of the APR of the contract and the rate-threshold.

The mathematical approach is an effective guide in order to make concrete the legal assumptions.

According to the instructions[31] provided by the Bank of Italy, the methodology of calculating the APR varies according to the different categories of operations.

The lease is in category 6), so the formula to calculate the A.P.R. is as follows:

$$\sum_{k=1}^{m} \frac{A_k}{(1+i)^{t_k}} = \sum_{k'=1}^{m'} \frac{A'_{k'}}{(1+i)^{t_{k'}}} \quad (1)$$

i is the APR;

k is the order number of a fee;

k' is the order number of a payment;

is the amount of the fee at the date k;

. is the amount of the payment at the date k';

m is the order number of the last loan;

m' is the order number of the last payment;

time interval between the first loan and additional loans from 2 to m;

. time interval between the first payment and additional payments from 1 to m';

Without loss of generality we can assume that the lessor entirely gives the loan to lessee at the beginning. The lessee, from his point of view, pays:

[31] Bank of Italy, Instructions for detecting average rates of charge under the Usury Act, 2009.

$$V = S + C + \sum_{k=1}^{n} \frac{C_k}{(1+i)^k} + \frac{P}{(1+i)^n} \qquad (2)$$

V is the initial amount;
S is the initial expense;
C is the first fee;
is the amount of the fee at the date k eventually increased by collection costs;
k is the order number of a fee;
i is APR;
n is the order number of the last payment;
P is the redemption price;
We report below some examples to clarify the concept[32]:
Example 1
On February 2, 2018 a lessor stipulates with a lessee a leasing contract at a specified time as follows:

Initial amount *V*	33.321,94679
initial expense S	363,25
first fee C	642,20
last fees	653,25
redemption price P	395,73
months	60
number of fees	60

We apply the equation (2), it follows:

APR	**8.5324%**

CATEGORY	REFERENCE PERIOD	APR	USURY THRESH-OLD-RATE
Car Leasing	January 1 st 2018 March 31 st 2018	8.5324%	12.0206%

[32] At this end it's useful to remember that the interest rate relative to a period equivalent to another previously assigned is: where k is

$$i_h = (1 + i_k)^{k/h} - 1$$

the number of period assigned and h is the number of the new period.

Comparing the value of APR and the value of the threshold-usurious rate must be concluded that in this lease transaction, the interest rate charged is not usurious.

Example 2

On February 2, 2018 a lessor stipulates with a lessee a leasing contract at a specified time as follows:

Initial amount V	33.883,1431
initial expense S	471,25
first fee C	698,27
last fees C_k	727,31
redemption price P	495,73
months	60
number of fees	60

We apply the equation (2), it follows:

APR	**13,0621 %**

CATEGORY	**REFERENCE PERIOD**	**APR**	**USURY THRESHOLD-RATE**
Car Leasing	January 1 st 2018 March 31 st 2018	13.0621 %	12.0206 %

Comparing the value of APR and the value of the threshold-usurious rate must be concluded that in this lease transaction, the interest rate charged is automatically usurious.

5. *The usury implemented through the internal discounting of future lease payments*

The clause internal discounting, through which a remuneration is agreed to the lessor, in the event of user's default, represents an agreed compensation, as well as a profitable or corresponding function.

Compared to the subject of discussion, the breach in the lease contract, represented by the failure or delay in repayment of capital, interest and accessories, leads to the creditor an consequential damage, represented to the loss of principal amount, and a lost profit, represented to the failure gain retracting of operation not regularly fulfilled.

It follows that the meaning of the criminal in the same contracts, is the compensation payable to the lessor for the financial loss in terms of payment of principal amount not returned and of failure to achieve the revenue expected from the exact fulfillment of the contract.

The clause «expired + expire - good» provides, in the event of contract termination, the payment of all accrued fees as well as those accruing until the natural end of the contract, including redemption, discounted at a rate determined.

It should be noted that the rate applied to operate such discounting is always much lower than that with which it is built the original amortization schedule and determined the interest component of amount of the lease payments.

In mathematical terms, it is true that the amount of the lease payments, discounted at the rate agreed (if positive), is less than the amount of such royalties which should have been paid to the original contractual deadlines. This amount is considerably higher than the sum of the principal amounts of all fees to expire, resulting for the user the payment of a part of the interests originally included in lease payments, however alleged by the lessor in advance and in a single solution, instead of the conventionally agreed deadlines.

It should determine, with the help of financial mathematics, if in these operations there is compliance with the rules in terms of wear, so as already briefly analyzed.

At this end it's useful to spin off the lessor credit[33]:

debt left;

the compensation received for damages;

penalty.

We consider that the lessee stops paying fees A_k at a certain date t_k and the lessor decides to stop the contract.

The lessor credit L_c (c indicates credit) is:

$$L_c = L(i) + R + \pi$$

where L(i) is the residual debt of the lessee as the amortization plan, R is the compensation for the lessor received for damages, and π is the penalty.

The debt left of the lessee L(i) is the outstanding fees and the redemption price A_n discounted with an interest rate (APR) contractually established:

$$L(i) = \sum_{s=k}^{n} A'_k \, (1+i)^{t_k - t_s}$$

[33] M. D'Amico, E. Luciano, L. Peccati, *Calcolo finanziario. Temi di base e temi moderni*, Milano, 2011, 278 ff.

We observe that the interest rate related to the period is the equivalent of the considered period[34].

The credit of the lessor calculates at the interest rate 0,3% as established on january 2018 is:

$$L(0{,}3\%) = \sum_{s=k}^{n} A'_k \, (1 + 0.3\%)^{t_k - t_s}$$

It's obvious that the rate interest $i_{0,3\%}$ must be appropriately converted (see footnote 34) according to the period of the fees.

To consider the spread between L_c and L(i) and to obtain the value of R and π we must examine the debt left L(i) and the compensation for damages R. In this case we consider a new reference interest rate i_{rr} to discount the fees A_k.

Therefore, we deduce $L(i_{rr})$, as follows:

$$L(i_{rr}) = L(i) + R$$

$$L(i_{rr}) = \sum_{s=k}^{N} A'_s \, (1 + i_{rr})^{t_k - t_s}$$

The rate follows the same indications for the calculation of L(i) and L_c.

Since it is:

$i_{0,3\%} < i_{rr} < i$

The present value is decreasing while the interest rate is increasing, so it follows:

$$L_c > L(i_{rr}) > L(i)$$

It is easy to deduce that the compensation for damage is:

$$R = L(i_{rr}) - L(i)$$

and the penalty is:

$$\pi = L_c - L(i_{rr})$$

To explain better, we consider the following example

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[34] It's useful to remember that the interest rate relative to a period equivalent to another previously assigned is : $i_h = (1 + i_k)^{k/h} - 1$

where k is the number of period assigned and h is the number of the new period.

On February 16, 2018 a lessor stipulates with a lessee a leasing contract at a specified time as follows:

Initial amount	24.745,63
initial expense	363,25
first fee	1890,25
last fees	189,25
redemption price	1815,75
months	24
number of fees	12
first fee not paid	6
APR	6%
legal rate	0,3%
reference interest rate	1,5%

We calculate the residual debt (equivalent APR 0.976 %):

$$(i) = 1890{,}25 + \frac{1890{,}25}{1{,}00976} + \frac{1890{,}25}{1{,}00976^2} + \cdots + \frac{1890{,}25 + 1815{,}75}{1{,}00976^6} = 14567{,}15$$

and the credit of the lessor (equivalent legal rate 0,0499376%)

$$L_c = 1890{,}25 + \frac{1890{,}25}{1{,}000499376} + \frac{1890{,}25}{1{,}000499376^2} + \cdots + \frac{1890{,}25 + 1815{,}75}{1{,}000499376^6} = 15022{,}2725$$

and the amount of the residual debt + compensation (reference interest rate 0,24804 %)

$$L(i_{rr}) = 1890{,}25 + \frac{1890{,}25}{1{,}0024804} + \frac{1890{,}25}{1{,}0024804^2} + \cdots + \frac{1890{,}25 + 1815{,}75}{1{,}0024804^6} = 14922{,}8978$$

So the compensation for the lessor of the leasing is:

$$R = 14922{,}8978 - 14567{,}15 = 355{,}7478$$

and the penalty:

$$\pi = 15022{,}2725 - 14922{,}8978 = 99{,}3747$$

Residual debt	L(i) = 14567,15
Compensation	355,7478
Penalty	99,3747
Total credit	L_c = 15022,2725

Finally we can prove that the APR of a leasing contract may be usurer before the implementation of the clause of the article 1526 c.c., but it becomes again lawful when we adopt the previous clause.

To explain we consider the following example:

On March 16, 2018 a lessor stipulates with a lessee a leasing contract at a specified time so we have:

Residual debt	L(i) = 79223,35
Compensation	13846,25
Penalty	1822,78
Total credit	L_c = 92562,889

$$i = \frac{L_c}{L(i)} - 1$$

We have APR= 16,83788%, that is it is an usury rate if to compare it with the usury thershold-rate (12.0206 %) relate to the period from January 1 st 2018 to March 31 st 2018.

APR	USURY THERSHOLD-RATE
16,83788%,	12.0206 %

However the rate becomes again lawful when we subtract the value of the good to the credit of the lessor as follow:

$$i = \frac{L_c - V}{L(i)} - 1$$

If we suppose that the value of the good is V=9869,55627 €, we obtain APR= 4,38 % and this strategy hides, clearly, the usury inherent the contract.

APR	**THERSHOLD-RATE**
4,38%	12,026%

6. *Conclusion*

The evolution of case law has highlighted the need of contractual provision of the user's right to impute the value of the leaseback to the sum due in returning rates to expire. This, in order to prevent to penalty clauses attributing the lessor excessive benefits.

The various judgments of the Supreme Court, greatly influencing practice, seemed to be able to balance the positions of lessor and lessee, in the event of breach of contract. In this event, they saw a strong first enrichment at the expense of the second. Subtracting the value of the sale of the good from the final credit, prevented the leasing company to exercise a strong position towards the contractor weak.

The case law, however, seems to have always worked without taking into consideration the wear discipline, or rather, without carefully studying the impact this new clause that could have had in defining the contractual interests.

In the present work, it has come to try as the subtraction's obligation of the value of good sale is really a chance for the leasing companies to hide the wear of the contract.

The statements of jurisprudence, in fact, apparently flawless, try to solve the problem reducing the due credit by the defaulting party (through the subtraction of the value of good sale), rather than upstream, by acting on the factors that lead to the determination of a credit: the sum of the remaining debt with compensation for damages and criminal.

In fact, the credit resulting from activation of the clause, is far higher than the outstanding debt as per amortization schedule. So, as, in credit are included, in addition to payment of the remaining debt, compensation for damages and criminal.

The legal rate is normally lower than the APR. This is a substantial increase of the compensation values of the damage and criminal. So the rate of game that we have seen – in the formula makes the APR of the contract, at the activation of the clause, but before the supply of goods, a usurious rate.

From the calculation of the APR notice as the lessors are able to put usurious rates (verifiable only after the occurrence of default) in the clause allowing subsequently the subtracting the value of good sale, but they return again lawful, once minus the value of the good.

This problem emerges only through the fruitful assistance of financial mathematics.

Moreover, in assessing the penalty, expressed in terms of excessiveness, the court must compare the advantage that it ensures the contractor fulfilled with the profit margin that it legitimately hoped to draw from the regular execution of the contract.

A viable solution would be to calculate the APR. At every moment of every situation covered by the contract, especially in the event of user's default.

Risoluzione di leasing traslativo per inadempimento dell'utilizzatore: un esempio di analisi matematica del diritto

I metodi quantitativi e la matematica finanziaria svolgono una importante funzione quale ausilio alla contrattualistica, in particolare agli strumenti di tipo economico-matematico. Il binomio diritto matematica potrebbe portare ad una feconda evoluzione del diritto, soprattutto negli aspetti economici degli istituti giuridici. In tema di leasing traslativo, nell'ipotesi di risoluzione per inadempimento dell'utilizzatore, si analizza la clausola che prevede la determinazione del credito della società di leasing in ragione della somma del valore attuale complessivo di tutti i canoni maturati, nonché di quelli maturandi sino al naturale termine del rapporto, detratto il corrispettivo della rivendita o del reimpiego in *leasing* del bene. L'elaborato cercherà di provare che l'interesse contrattuale, all'attivazione della clausola, è differente rispetto all'interesse che emerge dopo la sottrazione del valore del prezzo di vendita del bene da credito residuo.

Termination due to user's breach in the leaseback: an example of mathematical analysis of the law

Quantitative methods and financial mathematics play a key role in aiding contracts, especially economic-mathematical tools. The coupling of jurisprudence and mathematics could lead to a prolific evolution of the law, especially in the economic aspects of juridical institutions. As regards the leaseback, the termination of a contract due to a breach by the user is analysed with particular reference to the contractual clause which includes the determination of the credit due to the leasing company on the basis of the payment of all rental accrued and accruing until the natural end of the contract, deposit included, after deducting the retail costs for the sale or re-leasing of the goods. The work will attempt to prove that the contractual interest rate, when the clause is activated, is different from the interest rate detectable once the value of the sale of the goods is subtracted from the credit due.